How Much Did She See?

Du Châtelet's Commentary on Newton's *Principia*

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1732 Maupertuis, *Discours sur les différentes figures des astres avec une exposition des systèmes de MM. Descartes et Newton* (re-issued in a revised version in 1742; English translation in Keill, 1734)

1737 Expedition returns from Lapland: Maupertuis, *La Figure de la Terre déterminée par les observations de MM. de Maupertuis, Clairaut, Camus, Le Monnier, Outhier, Celsius au cercle polaire* (1738)

1738 Clairaut, "An Inquiry concerning the Figure of Such Planets as Revolve about an Axis, Supposing the Density Continually to Vary, from the Centre to the Surface" (in *Phil. Trans.*)

1740 D. Bernoulli, Maclaurin, and Euler monographs on the tides, submitted for Paris Prize Problem

1739- Le Seur and Jacquier , "Jesuit" ed. of *Principia*, with extensive commentary (by eds. & Calandrino),
1742 much of it reformulating Newton's work within the calculus; includes above monographs on the tides

1742 Maclaurin, *Treatise on Fluxions* (includes work on equilibrium figure of rotating bodies of fluid)

1743 Clairaut, *Théorie de la figure de la terre, tirée des principes de l'hydrostatique*

1743 Expedition returns from Peru: Bouguer, *La figure de la Terre* (1749)

1747- Euler, *Recherches sur le Mouvement des Corps Célestes en Général* and *Recherches sur la question*
1749 *des inégalités du mouvement de Saturne et de Jupiter* (Paris Prize, 1748)

1748 Euler, "Réflexiones sur l'espace et le temps"

1748 Bradley, announcement of the 18 year nutation of the Earth, apparently caused by lunar gravity

1749 d'Alembert, *Recherches sur la précession des equinoxes et sur la nutation de l'axe de la terre dans système Newtonien*

1749 Clairaut, lunar apsides: *Théorie de la lune, déduite du seul principe le l'attraction réciproquement proportionelle aux quarrés des distances* (1752)

1752 d'Alembert, *Essai d'une nouvelle théorie de la résistance des fluides*

1752 Euler, *Recherches sur les irrégularités du mouvement de Jupiter et de Saturne* (Paris Prize, 1752)

1753 Euler, *Theoria motus lunae, exhibens omnes eius inaequalitates*

1753 Mayer, "Novae tabulae motuum solis et lunae" (awarded part of Longitude Prize in 1770)

Contemporaneous Commentaries on the *Principia*

1702 Gregory, ***Astronomiae Physicae & Geometricae Elementa***

1702 Keill, ***Introductio ad Verum Physicam, seu Lectiones Physicae***

1713 Cotes, Editor's (polemical) preface to the 2nd edition of the *Principia*

1718 Keill, ***Introductio ad Verum Astroniam, seu Lectiones Astronomicae***

1720 'sGravesande, ***Physices elementa mathematica, experimentis confirmata. Sive, introductio ad philosophiam Newtonianam***

1728 Pemberton, ***A View of Sir Isaac Newton's Philosophy***

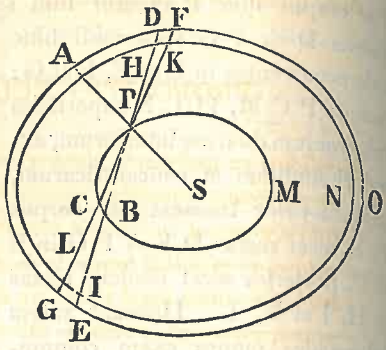
1738 Voltaire, ***Eléments de la philosophie de Newton***

1739-42 Le Seur, Jacquier, and Calandrino, a proposition by proposition commentary of the 3rd edition of the *Principia*

1746 Lacaille, ***Leçons élémentaires d'astronomie géométrique et physique***

1748 Maclaurin, ***An Account of Sir Isaac Newton's Philosophical Discoveries***

merarum similium concentricarum et axem communem habentium dividantur spatia D P F, E G C B in particulas, hæ omnes utrinque æqualiter trahent corpus P in partes contrarias. Æquales igitur sunt vires coni D P F et segmenti conici E G C B, et per contrarietatem se mutuo destruunt. Et par est ratio virium materiæ omnis extra sphæroidem intimam P C B M. Trahitur igitur corpus P a sola sphæroide intimâ P C B M, et propterea (per Corol. 3. Prop. LXXII.) attractio ejus est ad vim, quâ corpus A trahitur a sphæroide totâ A G O D, ut distantia P S ad distantiam A S. Q. e. d.



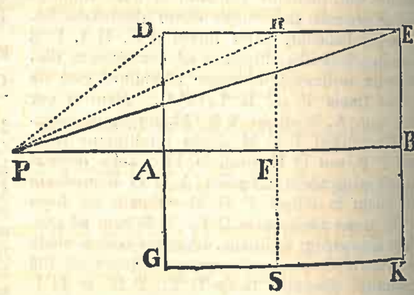
PROPOSITIO XCII. PROBLEMA XLVI.

Dato corpore attractivo, invenire rationem decrementi virium centripetarum in ejus puncta singula tendentium.

E corpore dato formanda est sphæra vel cylindrus aliave figura regularis, cujus lex attractionis, cuivis decrementi rationi congruens (per Prop. LXXX. LXXXI. et XCI.) (a) inveniri potest. Dein factis experimentis

(a) * *Inveniri potest.* Hoc est per Propositiones citatas inveniri potest generalis expressio seu formula attractionis corpusculi in sphæram vel cylindrum aliamve figuram regularem, et lex attractionis corpusculi in eandem figuram experimentis inventa conferri debet cum generali illâ formulâ, et indè habebitur æquatio cujus ope determinari poterit formulæ generalis exponens indeterminata, quæ exhibebit attractionem in singulas particulas materiæ.

Exemplum. In cylindrum A D E K G trahatur corpusculum P, situm in ejus axe A B, ut in Prop. XCI.; supponaturque vis in singulas cylindri particulas tendens reciprocè ut distantiæ dignitas cujus index n, et dicatur $PA = a$, $PD = b$, $PB = c$, $PE = e$, $RF = g$, $PF = x$, $PR = y$, eritque $yy = xx + gg$, ideoque $ydy = xdx$. Quare fluxio vis quâ corpusculum P in cylindrum A D R S G trahitur, erit (541) ut $\frac{dx}{x^{n-2}} - \frac{xdx}{y^{n-1}} = \frac{dx}{x^{n-2}} - \frac{ydy}{y^{n-1}} = x^{2-n}dx - y^{2-n}dy$; cujus fluens $= \frac{x^{3-n} - y^{3-n} + Q \text{ const.}}{3-n}$; hæc autem



evanescit, ubi $x = a$, et $y = b$; Quare erit $Q = b^{3-n} - a^{3-n}$, et fluens accurata $= \frac{b^{3-n} - a^{3-n} + c^{3-n} - e^{3-n}}{3-n}$, ubi $x = c$, et $y = e$. Jam verò vis quâ corpusculum P in totum cylindrum A D E K G trahitur, experimentis inventa sit ut $b - a + c - e$, et habebitur æquatio $b - a + c - e = \frac{b^{3-n} - a^{3-n} + c^{3-n} - e^{3-n}}{3-n}$, ex quâ determinandus est valor indicis generalis n. Porro posito $n = 2$, æqualia fiunt æquationis membra, ergo vis in singulas cylindri particulas tendens erit reciprocè ut quadratum distantiæ a particulâ, quemadmodum in Cor. 1. Prop. 91. positum est. Verum si hâc ratione, varios tentando numeros, non potest indicis generalis n valor inveniri, ponatur $3 - n = z$, et vis corpusculi in cylindrum experimentis reperta sit ut quantitas q; et erit $qz = b^z - a^z + c^z - e^z$. Fiat $a^z = p$, $b^z = v$, $c^z = r$, $e^z = s$. et erit (L significante Logarithmum quantitatis cui præfigitur) $L.a^z = L.p$, $L.b^z = L.v$, $L.c^z = L.r$, $L.e^z = L.s$, adeoque $zL.a = L.p$, et $z = \frac{L.p}{L.a} = \frac{L.v}{L.b} = \frac{L.r}{L.c} = \frac{L.s}{L.e}$. Unde $\frac{L.a \times L.v}{L.b} = L.p$, atque adeo $L.v^{\frac{L.a}{L.b}} = L.p$, proindeque $v^{\frac{L.a}{L.b}} = p$, et simili modo invenietur $v^{\frac{L.c}{L.b}} = r$, et $v^{\frac{L.e}{L.b}} = s$. Quare æquatio erit $\frac{q.L.v}{L.b} = v - v^{\frac{L.a}{L.b}} + v^{\frac{L.c}{L.b}} - v^{\frac{L.e}{L.b}}$, quæ ab exponente indeterminata libera est. Ut autem tollatur etiam L. v, ponatur $v = t + 1$, et (383) erit $L.v = L.\overline{t+1} = t - \frac{1}{2}tt + \frac{1}{3}t^3 - \frac{1}{4}t^4 + \frac{1}{5}t^5 -$, &c. in infinit. Si itaque in æquatione modo inventa loco v scribatur $t + 1$, et loco L. v series $t - \frac{1}{1}t^2 + \frac{1}{3}t^3 -$, &c. obtinebitur æquatio ab exponentibus et logarithmis indeterminatis libera, ex quâ per reversionem serierum invenietur valor quantitatis t, et inde reperietur L. v, atque per L. v habebitur valor indicis z, et inde valor ipsius n. Nam cum sit $z = \frac{L.v}{L.a}$, et $L.v = L.\overline{t+1}$, erit $z = \frac{L.\overline{t+1}}{L.a}$, et $n = 3 - z = 3 - \frac{L.\overline{t+1}}{L.a}$.

Si in æquatione vel quantitate exponentiali proposita, indeterminata z in solis quantitatum datarum exponentibus reperiretur, hæc æquatio vel quantitas superiori methodo posset ad aliam reduci numero terminorum finitam, in quâ nulla esset amplius exponens vel logarithmus indeterminata. Nam si $q = fa^z + gb^{2z} + hc^{4z} +$, &c., sitque $v = a^z$ erit $q = fv + gv^{\frac{2L.b}{L.a}} + hv^{\frac{4L.c}{L.a}} +$, &c. erit enim $z = \frac{L.v}{L.a}$ et $b^{2z} = b^{2\frac{L.v}{L.a}}$ et $L.b^{2z} = \frac{2L.v}{L.a} \times L.b = \frac{2L.b}{L.a}L.v$, unde est $b^{2z} = v^{\frac{2L.b}{L.a}}$ et sic de cæteris.

invenienda est vis attractionis in diversis distantiis, et lex attractionis in totum inde patefacta dabit rationem decrementi virium partium singularum, quam invenire oportuit.

PROPOSITIO XCIII. THEOREMA XLVII.

Si solidum ex unâ parte planum ex reliquis autem partibus infinitum, constet ex particulis æqualibus æqualiter attractivis, quarum vires in recessu a solido decrescunt in ratione potestatis cujusvis distantiarum plusquam quadraticæ, et vi solidi totius corpusculum ad utramvis plani partem constitutum trahatur: dico quod solidi vis illa attractiva, in recessu ab ejus superficie planâ, decrescet in ratione potestatis, cujus latus est distantia corpusculi a plano, et index ternario minor quam index potestatis distantiarum.

Cas. 1. Sit L G l planum quo solidum terminatur. Jaceat solidum

Du Châtelet's Access to Newton's Works

Principia Philosophiae Mathematica Naturalis

2nd edition: 1713

3rd edition: 1726

***De Mundi Systemate*:1728, reprinted 1731**

(i.e., the edited version of Newton's *De Motu Corporum, Liber Secundus*, described in a preface to it as Book 3, "composed in the popular style;" but actually composed in the middle of 1685, more than a year before Newton started on Book 3 "in the mathematical style")

"Theoria Lunae": 1702

published as an Appendix to D. Gregory's *Astronomiae Physicae & Geometricae Elementa*

Contents of the Du Châtelet Commentary

quelle ne peut être construite, comme il est aisé de le voir, que par l'opération du cas premier, où l'on a vû par la nature de la courbe, ainsi que par celle du Problême, que le corps en partant du point P s'éloignera de plus en plus du centre.

XXXII.

PROPOSITION XXI. PROBLÊME XI.

Trouver la trajectoire que le corps décrira en supposant $Y = \frac{n}{yy} + \frac{mn}{y^3}$.

On aura dans ce cas $\int Y dy = -\frac{n}{y} - \frac{mn}{2yy}$ en intégrant : alors l'équation générale trouvée (Article 17.) $dx = \frac{dy}{y\sqrt{\frac{2Byy - 2yy\int Y dy}{l^2f^2} - 1}}$ se changera en $dy = \frac{dy}{y\sqrt{\frac{2Byy + 2ny + nm}{l^2f^2} - 1}}$. Mais on a trouvé dans ce même article, $\frac{2B - 2\int Y dy}{l^2f^2} = \frac{1}{p^2}$, donc on aura $2B + \frac{2n}{h} + \frac{nm}{hh} = f^2$ (en mettant h pour y & l pour p) d'où on tire $2B = f^2 - \frac{2n}{h} - \frac{nm}{hh}$; donc $dx = \frac{lfdy}{y\sqrt{\left(f^2 - \frac{2n}{h} - \frac{nm}{hh}\right)yy + 2ny + nm - l^2f^2}}$.

Pour essayer de réduire cette équation aux équations polaires des sections coniques, je lui donne cette forme $dx = \frac{\frac{lfdy}{\sqrt{l^2f^2 - mn}}}{y\sqrt{\frac{\left(f^2 - \frac{2n}{h} - \frac{nm}{hh}\right)}{l^2f^2 - nm}yy + \frac{2ny}{l^2f^2 - nm} - 1}}$, d'où l'on tire

SECTION II.

TROISIEME PARTIE.

De l'attraction des sphéroïdes en particulier.

XXXVII.

PROPOSITION XVII. PROBLÊME XVII.

Trouver l'attraction qu'un sphéroïde BMO *exerce sur un corpuscule* A *placé sur son axe de révolution dans l'hypothèse que ses parties attirent en raison renversée du quarré de la distance.*

Je commence par faire les lignes $AB = f$. $BC = a$ = au demi axe du sphéroïde. $PB = x$. $PM = y$. $CD = b$ = au rayon de l'équateur, on aura par la propriété de l'ellipse $y = \frac{b}{a}\sqrt{2ax - xx}$; donc $AM = \sqrt{(f+x)^2 + yy} = \sqrt{\frac{bb}{aa}(2ax - xx) + ff + 2fx + xx}$.

Faisant à présent $n = -2$ dans la valeur $\frac{c}{r(n+1)}(AP \times AM^{n+1} - AP^{n+1})$ de l'attraction du cercle PM sur le corpuscule A trouvée (Article 22.) lorsque l'attraction est supposée agir comme une puissance n de la distance : on aura $\frac{c}{r}\left(1 - \frac{AP}{AM}\right)$ pour l'attraction du cercle PM dans la supposition présente, c'est-à-dire, que $\int\left(\frac{cdx}{r} - \frac{c(f+x)dx}{r\sqrt{\frac{bb}{aa}(2ax - xx) + ff + 2fx + xx}}\right)$ sera l'attraction cherchée.

Elements of Newton's Theory of Gravitational Attraction

1. **The force of gravity diminishes in an inverse-square proportion with distance.**

2. **The force is proportional to the mass of the body on which it acts.**

3. **The force is proportional to the mass of the body toward which it is directed.**

4. **Newton's third law of action and reaction holds for the forces of gravity – i.e. the force of gravity is mutual between any two bodies.**

5. **The force toward any body is composed out of forces toward its individual parts.**

6. **As a consequence of gravity, there is a *conatus* or tendency to accelerate toward every body that "fills the space" surrounding it.**

7. **There are mutual gravitational forces between every pair of particles throughout the universe.**

Du Châtelet on the Inverse-Square

From Kepler's 3/2 power rule, "the force that draws the planets toward the Sun decreases in the inverse ratio of the square of their distance from this celestial body, supposing that they turn in concentric circles about the Sun."

"The idea that presents itself most naturally to the spirit … is that they carry out their revolutions in concentric circles; but their apparent diameters, & more exact observations, made known long ago that their orbits could not be concentric to the Sun: therefore, before Kepler, their course was explained using eccentric circles which satisfied well enough the observations of the Sun & the planets, if we except Mercury & Mars."

"…[the Keplerian ellipse] agrees so perfectly with the Phenomena, that it is presently recognized by all Astronomers that it is in ellipses that the planets turn around the Sun, & that this celestial body occupies one of the foci of these ellipses."

"Starting from this discovery, M. Newton searched for the law of centripetal force that is necessary to make the planets describe an ellipse, & he found in prop. 11 that this force must follow the inverse proportion of the square of the body's distances from the focus of this ellipse; … there only remained, to be entirely certain that the centripetal force directing the celestial bodies in their course follows the inverse proportion of the square of the distances, to examine if the periodic times follow the same proportion in ellipses as in circles."

In Contrast to Newton and his *Principia*

PROPOSITIONES.

PROPOSITIO I. THEOREMA I.

Vires, quibus planetæ circumjoviales perpetuo retrahuntur a motibus rectilineis et in orbibus suis retinentur, respicere centrum Jovis, et esse reciprocè ut quadrata distantiarum locorum ab eodem centro.

PATET pars prior propositionis per phænomenon primum, et propositionem secundam vel tertiam libri primi: et pars posterior per phænomenon primum, et corollarium sextum propositionis quartæ ejusdem libri.

Idem intellige de planetis qui Saturnum comitantur, per phænomenon secundum.

PROPOSITIO II. THEOREMA II.

Vires, quibus planetæ primarii perpetuo retrahuntur a motibus rectilineis, et in orbibus suis retinentur, respicere Solem, et esse reciprocè ut quadrata distantiarum ab ipsius centro.

Patet pars prior propositionis per phænomenon quintum, et propositionem secundam libri primi: et pars posterior per phænomenon quartum, et propositionem quartam ejusdem libri. Accuratissimè autem demonstratur hæc pars propositionis (*) per quietem apheliorum. Nam

(*) * *Per quietem apheliorum.* * Astronomi motus coelestes calculant referendo astra ad eclipticam, cujus initium per intersectionem æquatoris et eclipticæ determinatur; sed illud initium fixum non est, et propter axis Terræ nutationem intersectio illa in antecedentia fertur 51 circiter secundis singulo anno, hinc fixæ toti-dem secundis progredi videntur. Aphelia planetarum etiam progredi videntur respectu ejus initii eclipticæ, progreditur ergo singulo anno.

Aphelium	Terræ	62".
	Saturni	78".
	Jovis	57".
	Martis	72".
	Veneris	86".
	Mercurii	80".

Sed multum abest quàm ut ille apheliorum motus, certissime determinetur, et uniformis esse deprehendatur, ex observationibus motûs aphelii Terræ nunc plus procedere quàm 50" nunc minus deprehenditur, unde quidam astronomi non alium esse ejus motum præter motum ipsius initii eclipticæ censent. Pariter ex observationibus aphelii Saturni, ejus motus irregularis videretur, aliquando accelerari, aliquando retrocedere, ex. gratia, ab anno 1694 ad finem anni 1708, minutis ferè 53 retrocessisse testatur Cassinus. Aphelium Jovis ad motum fixarum proximè accedere videtur, &c. Unde constat, aphelia quamproximè quiescere, et eam quantitatem exiguam motûs ipsis assignati quæ excedit motum fixarum, forte observationum erroribus deberi,

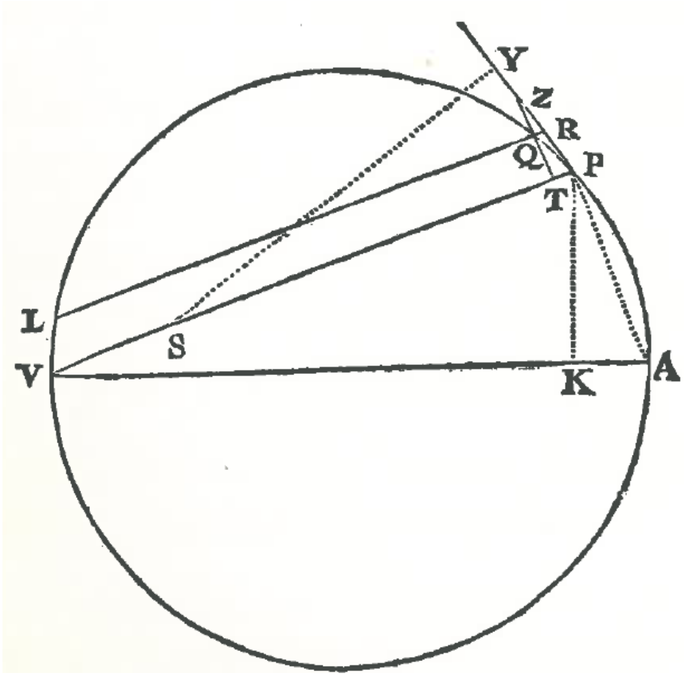
Inferences from phenomena are licensed by Theorems of the form,

IF
P *quampromixé*
THEN
Q *quamproximé*

Q then *taken* to be either *accuraté* or *quamproximé* true "until yet other phenomena make such propositions either more *accuraté* or liable to exceptions."

The most telling evidence, then, to come from "residual phenomena" – i.e. discrepancies between theory and observation.

Why Not Infer the Inverse-Square from Kepler's Ellipse *Quamproximé* ?



Because, as Newton knew already in 1684, in an eccentric circle indistinguishable from a Keplerian ellipse – as with Venus, Jupiter, and Saturn and their small eccentricities ε – the centripetal force does not vary as $1/SP^2$ *quamproximé*, but rather as $1/(SP^2 \times PV^3)$; that is, inversely as

$$\left(\frac{SP}{dia}\right)^5 + 3(1-\varepsilon^2)\left(\frac{SP}{dia}\right)^3 + 3(1-\varepsilon^2)^2\left(\frac{SP}{dia}\right) + (1-\varepsilon^2)^3\left(\frac{SP}{dia}\right)^{-1}$$

What Newton Actually Says About the Keplerian Ellipse

About the Sun, librated in this way, the other Planets revolve in Elliptical Orbits and, by radii drawn to the Sun, describe areas proportional to the times very nearly [*quamproximé*], as has been explained [*expositum*] (in Prop. 65). If the Sun were at rest and the Planets did not act on one another, the Orbits would be Elliptical and the areas would be proportional to the times exactly [*exacté*] (by Prop. 11 and Prop. 13, Corol. 1)….

If the Sun were at rest and the Planets had no action among one another [*in se invicem*], their Aphelia and Nodes would also be at rest (by Prop. 1 and Prop. 13, Corol. 1), and the major axes of their Elliptical orbits would be as the cube roots of the squares of their periodic times (by Prop. 15), and thus would be given from their given periodic times…. Moreover, Astronomical Observations appear to confirm that the Aphelia advance very slowly [*tardissime*] and the Nodes regress with respect to the fixed stars.

as per *De Mundi Systemate*, 1728

Newton's Goal for Natural Philosophy

Sic etiamsi colores ad Physicam pertineant, eorum tamen scientia pro Mathematica habenda est, quatenus ratione mathematica tractantur. Imo vero cum horum accurata scientia videatur ex difficillimis esse quae Philosophus desideret; spero me quasi exemplo monstraturum quantum Mathesis in Pjhilosophia naturali valeat; et exinde ut homines Geometras ad examen Naturae strictius aggrediendum, & avidos scientiae naturalis ad Geometriam prius addiscendum horter; ut ne priores suum omnino tempus in speculationibus humanae vitae nequaquam profuturis absumant, neque posteriores operam praepostera methodo usque navantes, a spe sua perpetuo decidant: Verum ut Geometris philosophantibus & Philosophis exercentibus Geometriam, pro conjecturis et probabilibus quae venditantur ubique, scientiam Naturae summis tandem evidentiis firmatam nanciscamur.

***Optical Lectures*, Lect. 3, 1670-72**

…Thus although colors may belong to physics, the science of them must nevertheless be considered mathematical, insofar as they are treated by mathematical reasoning. Indeed, since an accurate science of them seems to be one of the most difficult that philosophy is in need of, I hope to show – as it were, by my example – how valuable mathematics is in natural philosophy. I therefore urge geometers to investigate nature more rigorously, and those devoted to natural science to learn geometry first. Hence the former shall not entirely spend their time in speculations of no value to human life, nor shall the latter, while working assiduously with a preposterous method, perpetually fall short of their goal. But truly with the help of philosophical geometers and geometrical philosophers, instead of the conjectures and probabilities that are being blazoned about everywhere, we shall finally achieve a science of nature supported by the greatest evidence.

Why Did She Choose to Misrepresent the *Principia*?

- **She did not understand why Newton did not derive the inverse-square from the Keplerian ellipse, and thought he should have?**
- **She simply followed others (including even Leibniz in his *Tentamen*) in deriving the inverse-square from the Keplerian ellipse without worrying about why he didn't?**
- **She became persuaded by the derivations of the inverse-square from the Keplerian ellipse by a couple of individuals close to Newton that it was fully appropriate to do so?**
- **She thought Newton did not do so because astronomers had not yet then established the Keplerian ellipse from observations, but they had done so in the intervening years?**

Regardless, she seems not to have appreciated the extent to which Newton was trying to pursue a method in empirical research different from any that had gone before in natural philosophy.

Newton on $F_{\text{GRAV}} \propto$ Mass of Attracting Body

In the *Principia*:

Stipulate that the third law of motion holds for F_{GRAV}

Derive from celestial phenomena and experiment that $F_{\text{GRAV}} \propto$ mass of the attracted body

Infer that $F_{\text{GRAV}} \propto$ mass of the attracting body

As a corollary conclude that F_{GRAV} toward a body composed of inverse-square gravitational forces toward its parts

In *Liber Secundus*:

Note the agreement [*analogiam*] between F_{GRAV} and the size of the attracting planets

Derive from celestial phenomena and experiment that $F_{\text{GRAV}} \propto$ mass of the attracted body

"And since the action of centripetal force upon the attracted [*attractum*] body, at equal distances, is proportional to the matter in this body, it is reasonable also to grant [*rationi etiam consentaneum est*] that it is proportional as well to the matter in the attracting [*trahente*] body."

Du Châtelet on $F_{GRAV} \propto$ Mass of Attracting Body

From the mutual attraction of Jupiter and Saturn at conjunction and the attraction of the Moon on the Earth, as shown by be tides and the precession of the equinoxes, "we can therefore conclude that the attractive force belongs to all the celestial bodies."

Noting the agreement between F_{GRAV} and the size of the attracting planets, but "seeing as the size & mass are two different things, in order to be sure that gravity follows the law of masses, it was therefore necessary to know these masses."

"Because the attraction of all the celestial bodies that surround them follows the inverse-proportion of the square of the distances, it is quite likely that the parts of which they are composed attract each other in the same proportion."

Invoking Props. 74-76, "the total force of a planet is composed of the attractive force of its parts: for if we imagine that several little planets unite to make a big one, the force of this big planet would be composed of the forces of all these little planets."

Newton and Euler on this Question

***Prop. 92*: Given an attracting body, it is required to find the ratio by which the centripetal forces tending toward each of its individual points decrease.**

From the given body a sphere or cylinder or other regular figure [including a spheroid] is to be formed, whose law of attraction – corresponding to any ratio of decrease – can be found by props. 80, 81, and 91. Then, by making experiments, the force of attraction at different distances is to be found; and the law of attraction toward the whole that is thus revealed will give the ratio of the decrease of the forces tending towards each of its individual parts.

"It is still not decided by any single phenomenon that the attractive forces of heavenly bodies are proportional to their masses. On the contrary, Newton tried to determine the masses on this basis since there is no other way of specifying them. As soon as one now places the statement that the attractive forces are proportional to the masses (which is founded on a crude hypothesis) in doubt, …"

Euler to Mayer, Dec. 1751

Du Châtelet on $F_{GRAV} \propto$ Mass of Attracted Body

"But if the effect of the attraction, or the path made by the attracted body, depends on the mass of the attracting body, why would it not also depend on the mass of the attracted body?" [She here invokes Boyle's experiment with a feather and gold in a "vacuum"and Newton's double-pendulum experiment.] "It is therefore beyond doubt that the attractive force of our Earth proportions itself [*se proportionne*] to the mass of the body it attracts."

"…. so, the Sun attracts each planet in the direct ratio of its mass. The regularity of the orbit of the satellites of Jupiter around this planet is another proof of this truth, for M. *Newton* proved, Prop. 65, Cor. 3, that when a system of bodies moves in circles or in regular ellipses, it must be that these bodies experience no sensible action besides the attractive force that makes them describe these curves; … so if any of Jupiter's satellites, or Jupiter itself, was more attracted by the Sun than another satellite relative to its mass, then this greater attraction of the Sun would disturb the orbit of this satellite."

"Seeing as the attraction proportions itself to the mass of the attracting body, & to that of the attracted body, we must conclude from this that the attraction belongs to each part of the matter, & that all parts of which a body is composed attract each other mutually."

Du Châtelet on Newton's "Deduction" of the Precession of the Equinoxes

- **By analogy with his confirmed deduction of the mean motion of the lunar nodes: deduce the action of the Sun on an excess ring of mass around the equatorial Earth.**
- **"M. *Newton* gives thus the mean quantity of the motion of the equinoctial points. But it is not without examining the different varieties of the action of the Sun on the protuberance of the Earth at the equator, always by using the consideration of this ring…. We see by this that the axis of the Earth must change its position with respect to the ecliptic two times during its annual course & return twice to the same position."**
- **Now add the action of the Moon, "which is to that of the Sun as 4.4815 [51/3 in *Lib. Sec.*, 61/3 in 1st ed.] to 1 approximately," yielding 50″0‴12iv, "which is more or less, as we see, the quantity that the best observers have determined it to be."**

Du Châtelet on the Flux and Reflux of the Sea

- **"It was easy to notice … that these phenomena depend on the position on the Earth with respect to the Sun & to the Moon, but it was not easy to know the manner in which these two celestial bodies produce them, & the quantity that either one contributes. We only see the effects in which these actions are so intermingled, that without M. *Newton's* principle we would not have been able to untangle the one from the other, nor to assign their quantity."**
- **"…the force of the Sun on the waters of the sea is to that of the Moon, as 1 to 4½ approximately."**
- **"M. *Daniel Bernoulli* adds that the heights of the tides in the ports where observations are made depend on some many accidental circumstances that they cannot be exactly proportional to the heights of the tides far at sea. …[He] concludes that it would be surer to evaluate the respective forces of the Sun & of the Moon on the tides by their duration & their intervals rather than by their heights, & by using this method finds that the force of the Moon is in a lesser proportion to that of the Sun than the one M. *Newton* found." [namely 2½ to 1]**

Du Châtelet on the Figure of the Earth

- **Newton deduced that, if the Earth is in hydrostatic equilibrium and gravity toward it arises from inverse-square gravity toward its individual parts, then, the ratio of its polar diameter to its equatorial diameter is 229/230, and surface gravity decreases from its pole to its equator by 0.21 percent, so long as its density is uniform.**

- **Newton further proposed that, if the decrease in gravity is greater than this, the Earth's density increases toward the center and its *flatness* is greater than 1/230.**

- **Clairaut (1738): to the contrary, a flatness of 1/230 is a maximum under the stated assumptions, and that, if the decrease in gravity from pole to equator is greater than 0.21 percent, the density does indeed increase toward the center, but its flatness is less than 1/230.**

- **Clairaut (1743): if the Earth is in hydrostatic equilibrium and gravity toward it arises from inverse-square gravity toward its individual parts, then the Earth is a spheroid and there is a systematic relationship between its *flatness* and the decrease in surface gravity from pole to equator**

$$\frac{g}{g_{equator}} = 1 + \left(\frac{5}{2}m - f\right)\sin^2\phi + \ldots$$

- **Assuming that the Earth is in hydrostatic equilibrium, then the claim that its gravity arises from inverse-square gravity toward its parts, and with it Newton's claim of universal gravity, can be tested by determining whether "Clairaut's theorem" holds for it.**

Du Châtelet on the "Test"

- **"This great question of the figure of the Earth depends on the law according to which primitive gravity acts. … We were obliged to go measure a degree beneath the equator, & another beneath the polar circle, to decide this question."**

- **"The measures taken in Lapland & in Peru give a greater flattening than the one that we have just seen results from Newton's theory, for these measures give the ratio of the axes of 173 to 174."**

- **"It follows from M. *Clairaut*'s theory, that by admitting the suppositions that he makes on the interior of the Earth as the most natural among those that present themselves to the spirit, that the flattening can never be greater than 229 to 230, seeing as this ratio is the one we find while supposing the Earth to be homogeneous, & that it results from this theory that, in all other cases of the gravity increasing, the flattening must be less."**

- **"… in the experiments that have been carried out since M. *Newton* on the length of the pendulums in the different regions of the Earth, prove that these differences must not be attributed to this cause [thermal expansion from temperature differences], & that there really is a decrease in gravity from the pole to the equator greater than the one that M. *Newton* gave in his table."**

Du Châtelet on Newton on the Motion of the Moon

- **"The different kinds of motion that we had noticed long ago in the Moon, & the laws of these motions found by famous Astronomers, gave M. *Newton* the means of applying with success his theory to this planet. This great man, who had already made so many discoveries in the other parts of his System of the World, still wanted to perfect this one; & although the method he followed at this occasion is less clear & less satisfying than the one he had used for the other phenomena, we cannot prevent ourselves from owing him much recognition for having applied himself to it."**

- **"M. *Newton*, after having exposed the method by which he calculates the Moon's inequality that is called the variation, & the method he follows while determining the motion of the nodes, & the variation of the obliquity from the ecliptic, gives an account of what he says he has drawn from his theory of gravitation with regards to the other inequalities of the Moon…. In the examinations of the first inequalities, although the reader is not extremely satisfied because of a few suppositions & of a few abstractions made to make the problem easier [e.g a circular orbit], there is at least this advantage, that he sees the path of the Author & he acquires new principles with which he can flatter himself to go further. But as to that which regards the motion of the apogee & the variation of the eccentricity [i.e. the "evection"], & of all the other inequalities of the motion of the Moon, M. *Newton* contents himself with the results that are convenient for the Astronomers in the construction of tables of the Moon's motion, & he assures that his theory of gravity led him to the results."**

Du Châtelet's Parting Words on the Moon's Motion

- **"But how did M. *Newton* use these alterations to the central force, & which principles did he follow to avoid or to conquer the extreme complexity, & the difficulties of the calculation presented by this research? This is what we have not yet been able to discover at least not in a satisfactory way.**
  We find, I admit, in the first Book of the Principles, a proposition on the general motion of the apsides, which promises initially great usages for the theory of the apsides of the Moon, but when we come to use it, we soon see that it does not advance us much in this research."

- **"…we cannot without new contrivances that could be as difficult to find as the entire determination of the orbit of the Moon, use M. *Newton*'s general proposition on the apsides for the case of the Moon. Also, on this article as one the rest of the theory of the Moon, the greatest Geometers of this century abandoned the path beaten by M. *Newton*'s commentators up to the present, & believed that they would arrive earlier at the goal by retaking the whole work starting right from its origin. They looked to determine directly the paths & the speeds of any three given bodies that attract each other. We flatter ourselves in soon seeing the success of their work: the analytic method that they follow seems to be the only one that could really satisfy a research of this nature."**